

Dynamic Causal Modelling for evoked responses

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Overview

- 1 DCM: introduction
- 2 Neural ensembles dynamics
- 3 Bayesian inference
- 4 Conclusion

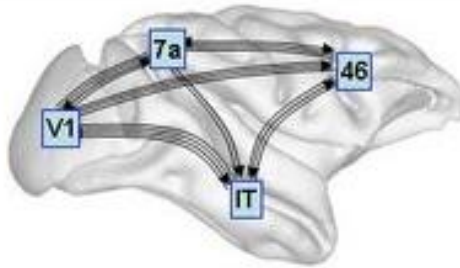
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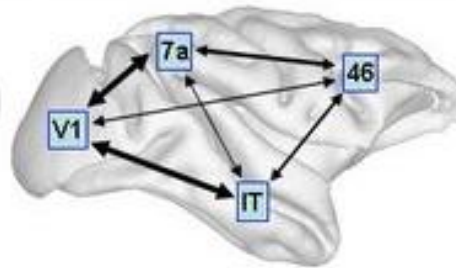
DCM: introduction

structural, functional and effective connectivity

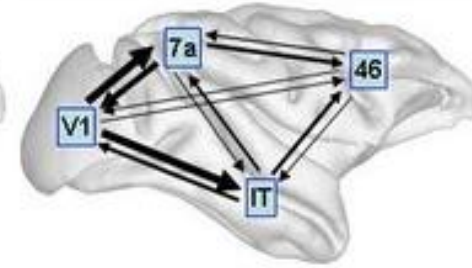
structural connectivity



functional connectivity



effective connectivity

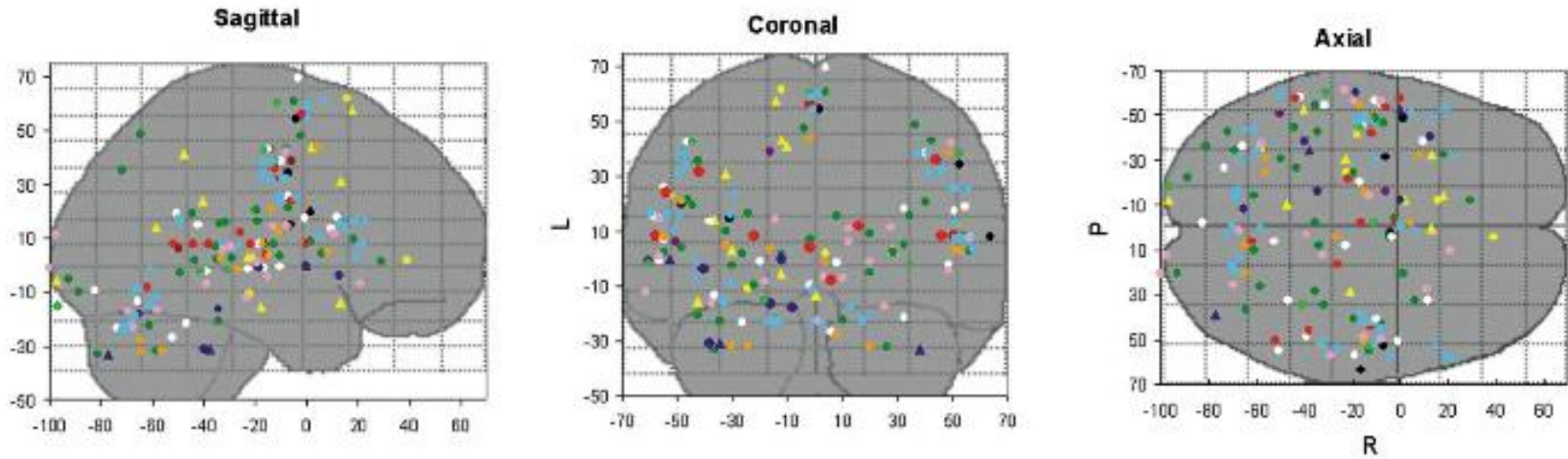


- ***structural* connectivity**
= presence of axonal connections
- ***functional* connectivity**
= statistical dependencies between regional time series
- ***effective* connectivity**
= causal (directed) influences between neuronal populations

DCM: introduction

connections are recruited in a *context-dependent* fashion

- meta-analysis on single-word reading (Turkeltaub, 2002)

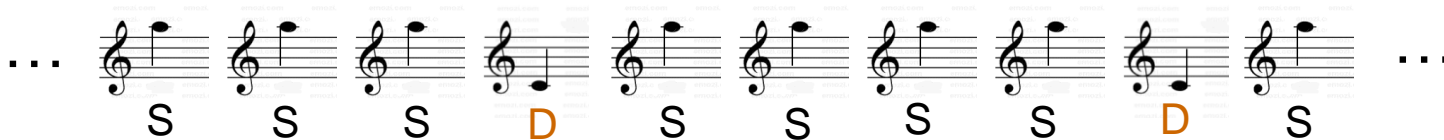


	Paper	Task	n	Within-Plane Res. (mm)	Between-Plane Res. (mm)	Filter (mm)	Critical Threshold	Foci
1	Peterson et al, 1988	read vs silent read	17	18	-	-	$p < .03$	8
2	Howard et al, 1992	read vs. falsefont aloud ("crime")	12	8	8.5	20	$p < .001$	2
3a	Price et al, 1994	read vs aloud false font feature det. (1000ms)	6	8	8.5	20	$p < .001$	3
3b		read vs aloud false font feature det. (150ms)						11
4	Bookheimer et al, 1995	read vs. random line drawing viewing	16	6.5	-	$5^3 \times 10$	$p < .001$	33
5	Price et al, 1994a	read vs. rest (1000ms)	6	6	8.5	20	$p < .001$	20
6	Price et al, 1994b	read vs rest (40 wds)	6	8	8.5	16	$p < .001$	12
7a	Herbstler et al, 1997	read irregular vs. aloud letter string ("hya")	10	-	-	16	$p < .001$	5
7b		read regular vs. aloud letter string ("hya")						3
8	Ramsay et al, 1997	read vs. fix (low freq, irregular)	14	6.5	5.5	$20^2 \times 12$	$p < .001$ & > 8 voxels	14
9	Jernigan et al, 1998	read (normal and degraded) vs fix	8	8.5	4.0	16	cor. $p < .06$ (2 or silent)	8
10a	Fiez et al, 1999	read vs fix (high freq consistent)	11	17	-	-	$p < .0006$	10
10b		read vs fix (high freq inconsistent)						9
10c		read vs fix (low freq consistent)						9
10d		read vs fix (low freq inconsistent)						11
11	Hagoort et al, 1999	read vs silent read (German)	11	9	9	18	$p < .05$ & > 40 voxels	17
								172

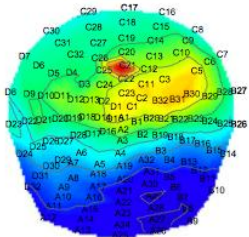
Introduction

DCM for evoked responses: auditory mismatch negativity

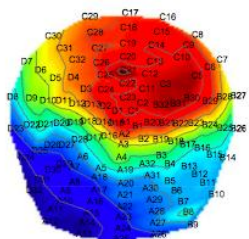
sequence of auditory stimuli



standard condition (S)

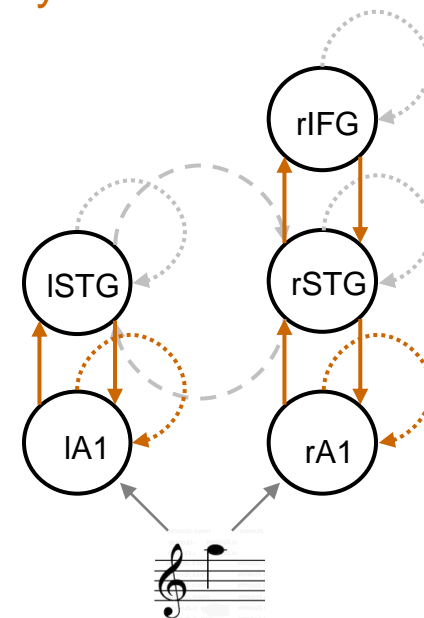
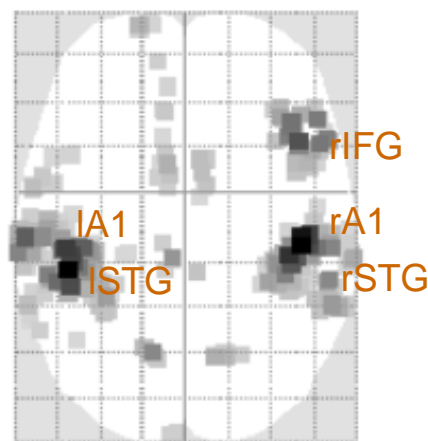


deviant condition (D)



t ~ 200 ms

S-D: reorganisation
of the connectivity structure



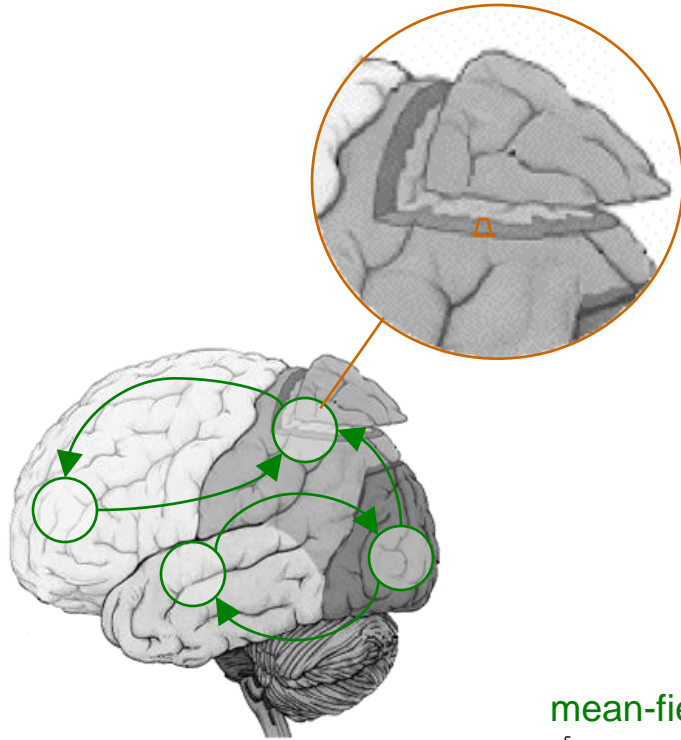
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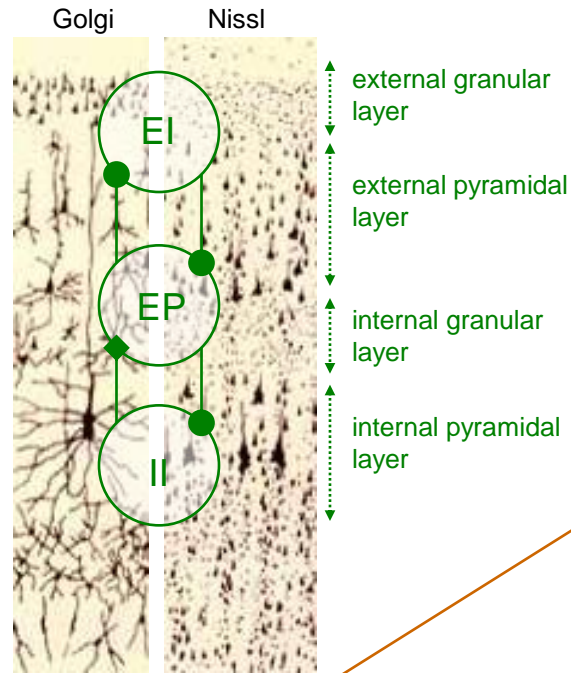
Neural ensembles dynamics

systems of neural populations

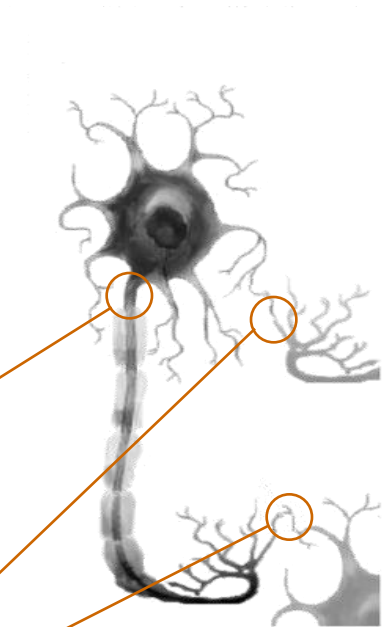
macro-scale



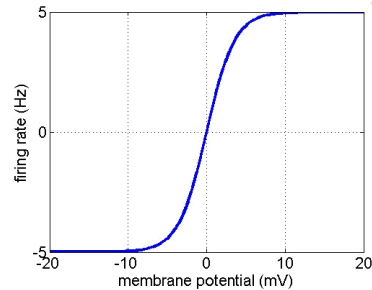
meso-scale



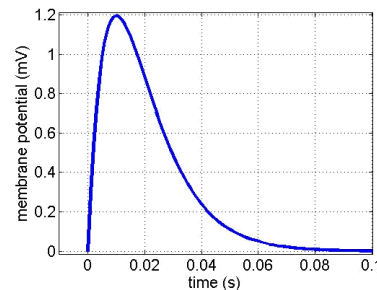
micro-scale



mean-field firing rate

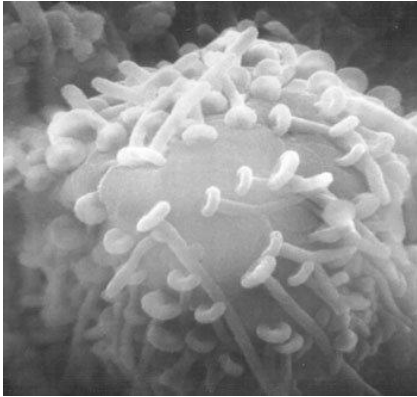


synaptic dynamics



Neural ensembles dynamics

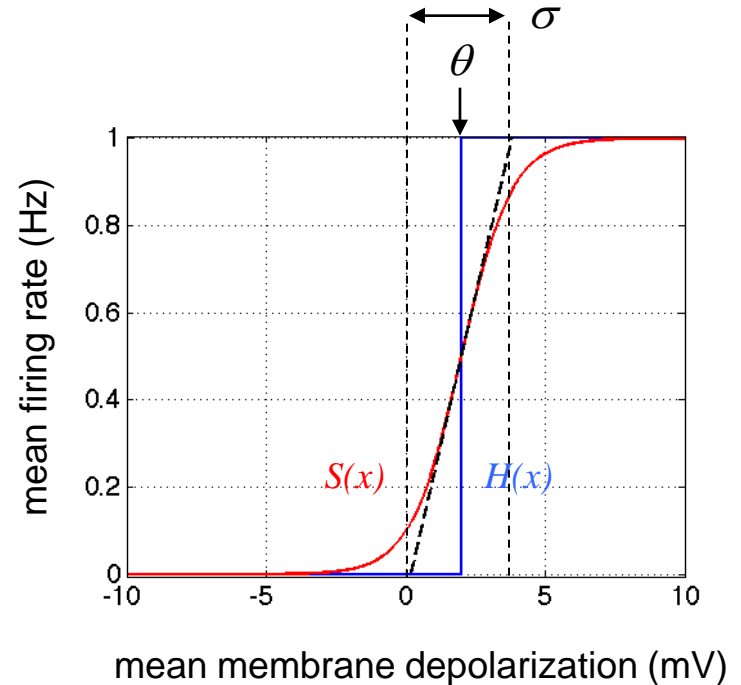
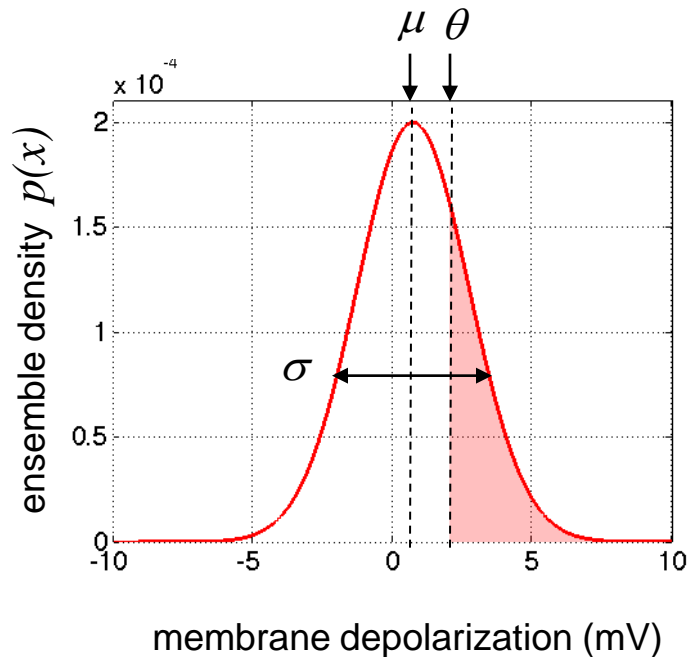
from micro- to meso-scale: mean-field treatment



x_j : post-synaptic potential of j^{th} neuron within its ensemble

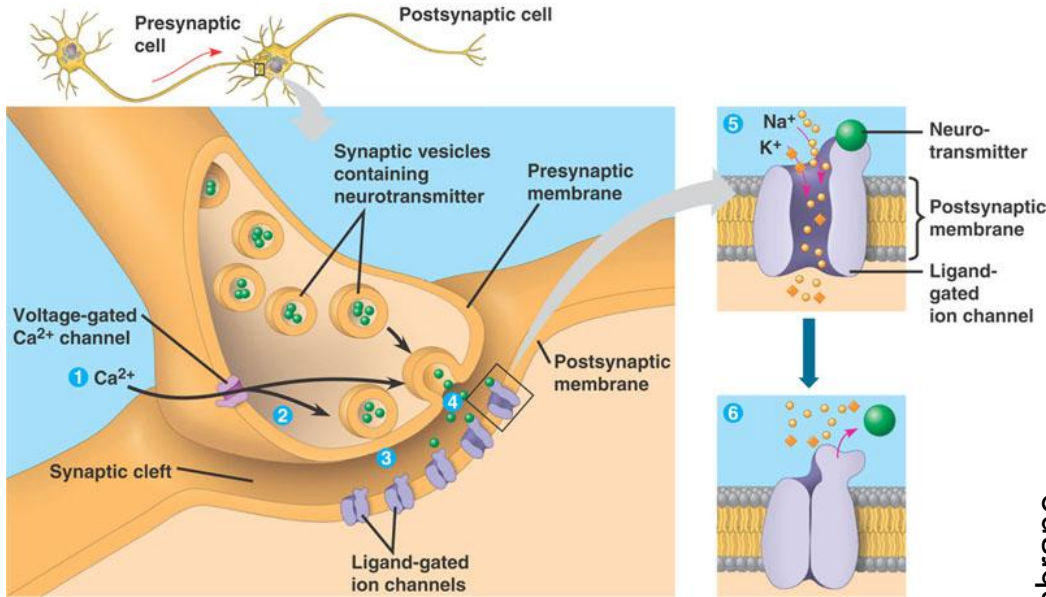
$$\frac{1}{N-1} \sum_{j \neq i} H(x_j, -\theta) \xrightarrow{N \rightarrow \infty} \int H(x - \theta) p(x) dx$$

$$= \int_{\theta}^{\infty} p(x) dx \approx S(\mu) \quad \text{mean firing rate}$$

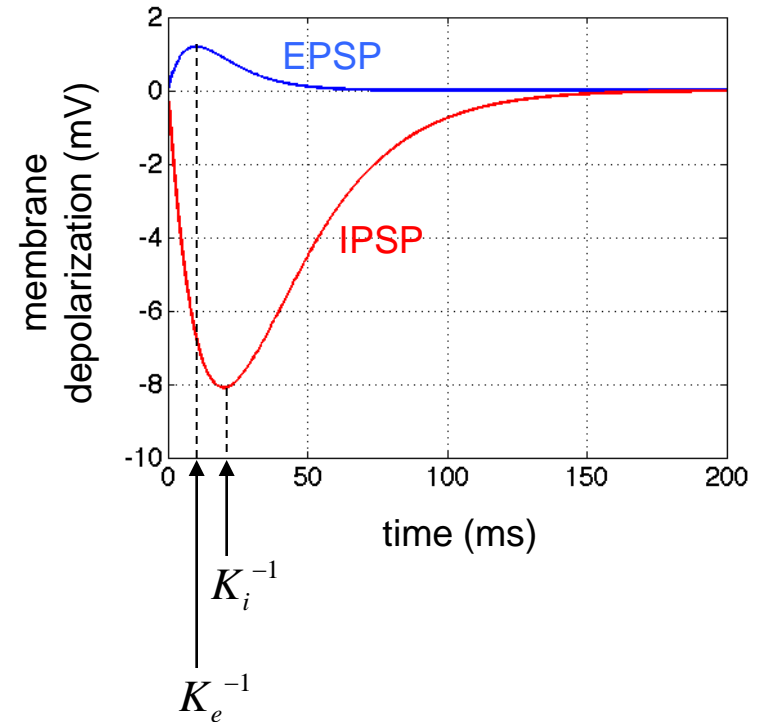


Neural ensembles dynamics

synaptic dynamics



post-synaptic potential

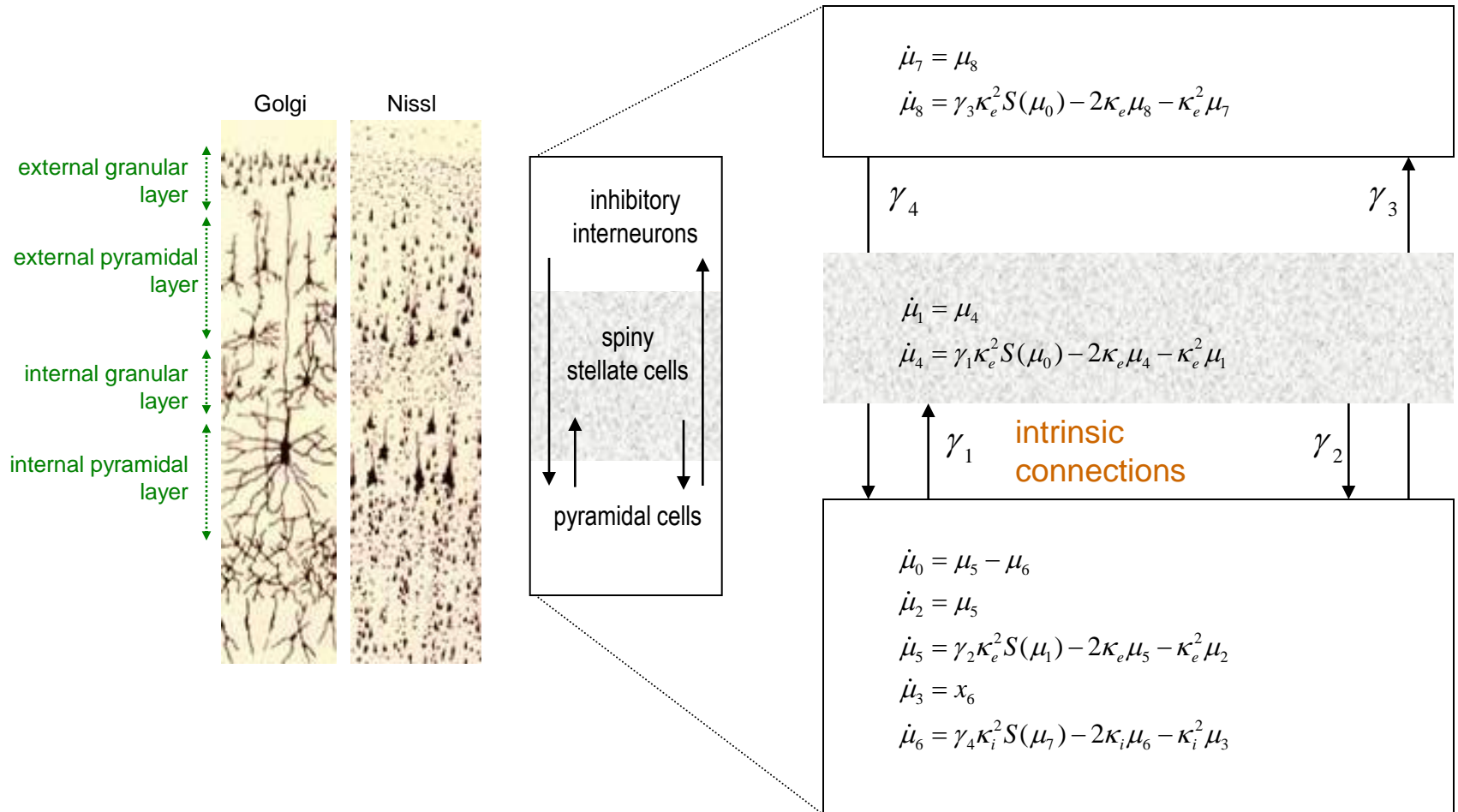


$$\mu(t) = S(u(t)) \otimes \text{kernel}_{PSP}(t)$$

$$\Leftrightarrow \begin{cases} \dot{\mu}_1 = \mu_2 \\ \dot{\mu}_2 = \kappa_{i/e}^2 S(u) - 2\kappa_{i/e} \mu_2 - \kappa_{i/e}^2 \mu_1 \end{cases}$$

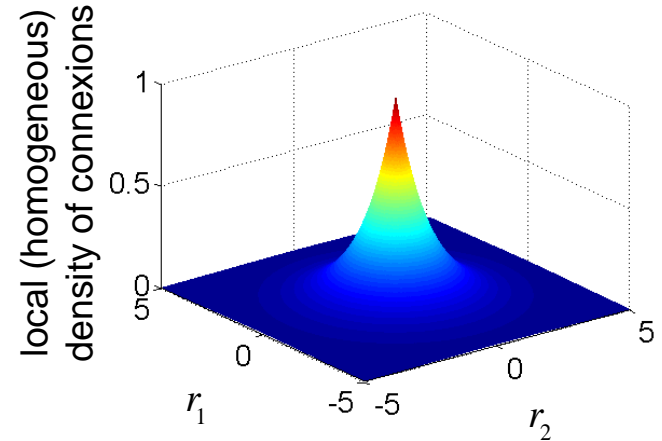
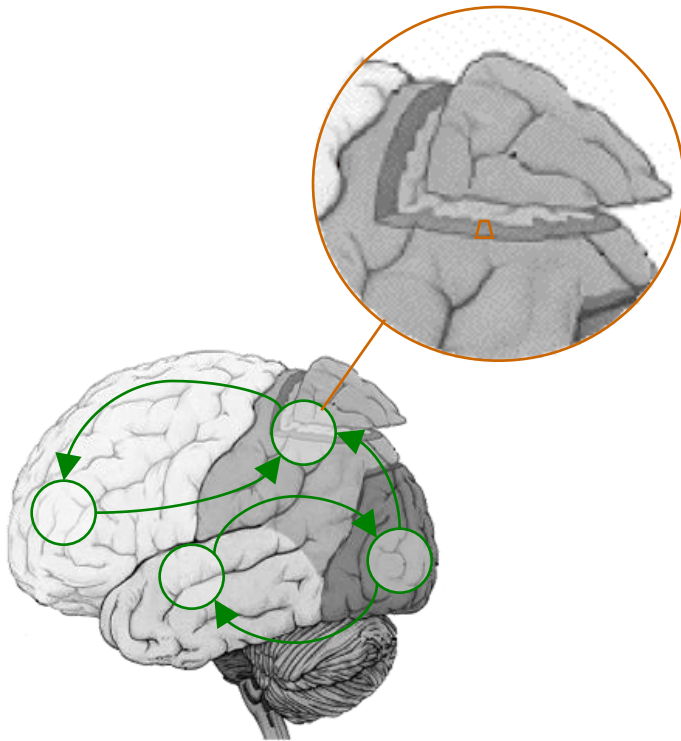
Neural ensembles dynamics

intrinsic connections within the cortical column



Neural ensembles dynamics

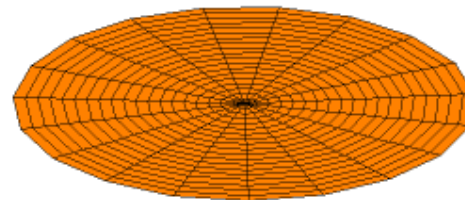
from meso- to macro-scale: neural fields



local wave propagation equation:

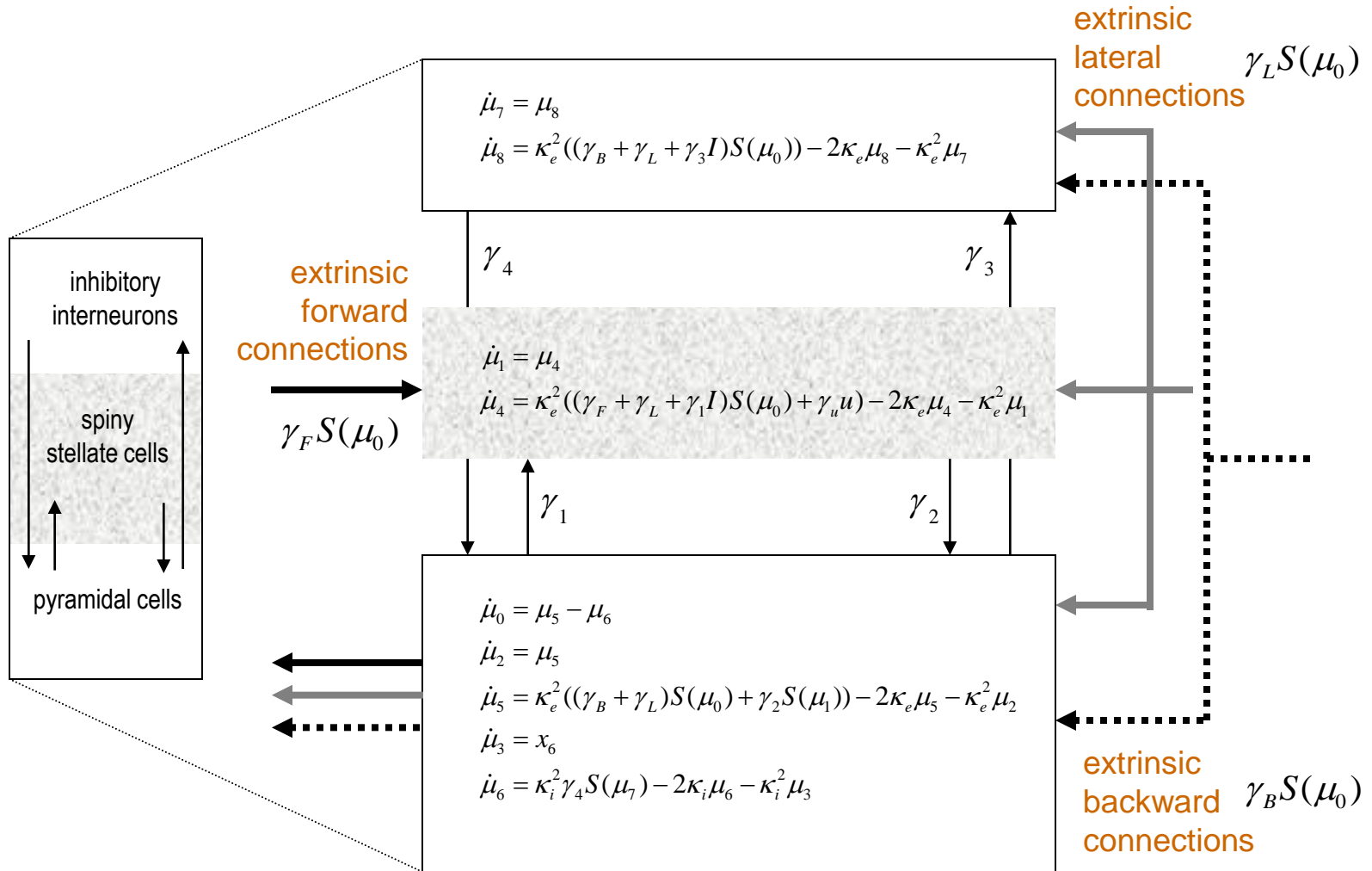
$$\left(\frac{\partial^2}{\partial t^2} + 2\kappa \frac{\partial}{\partial t} + \kappa^2 - \frac{3}{2} c^2 \nabla^2 \right) \mu^{(i)}(\mathbf{r}, t) \approx c\kappa \zeta^{(i)}(\mathbf{r}, t)$$

$$\zeta^{(i)} = \sum_{i'} \gamma_{ii'} S(\mu^{(i')})$$



Neural ensembles dynamics

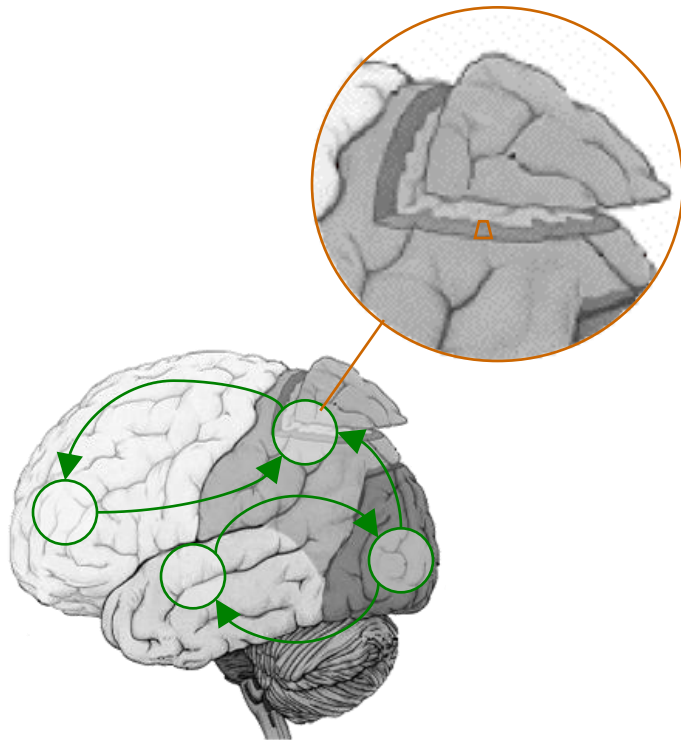
extrinsic connections between brain regions



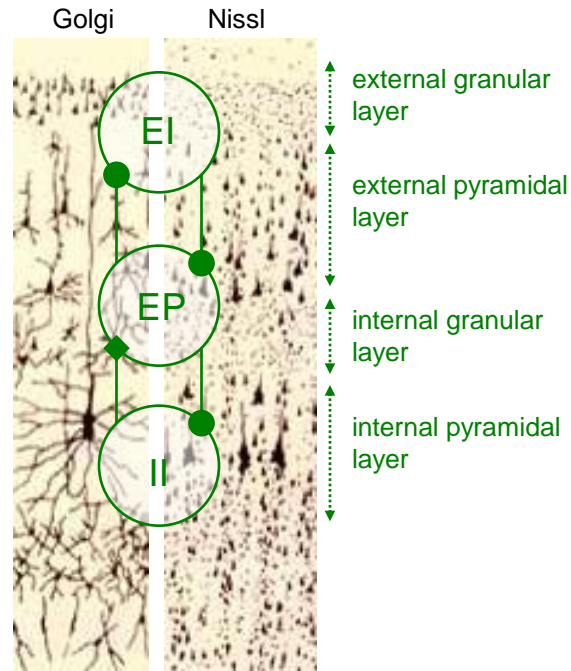
Neural ensembles dynamics

systems of neural populations

macro-scale



meso-scale



micro-scale

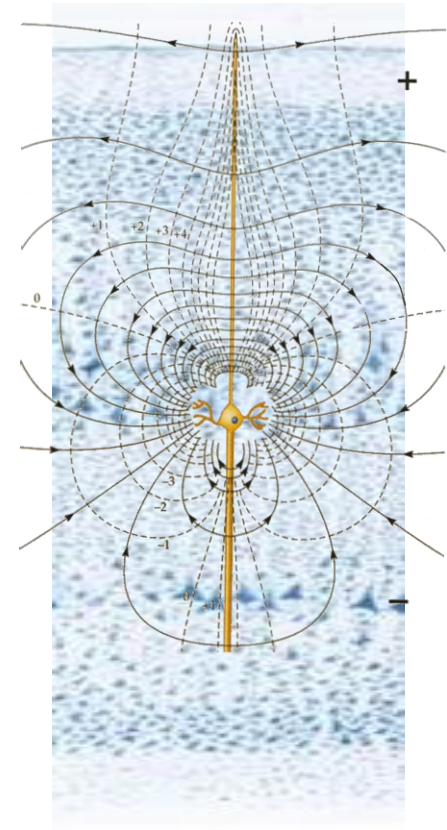
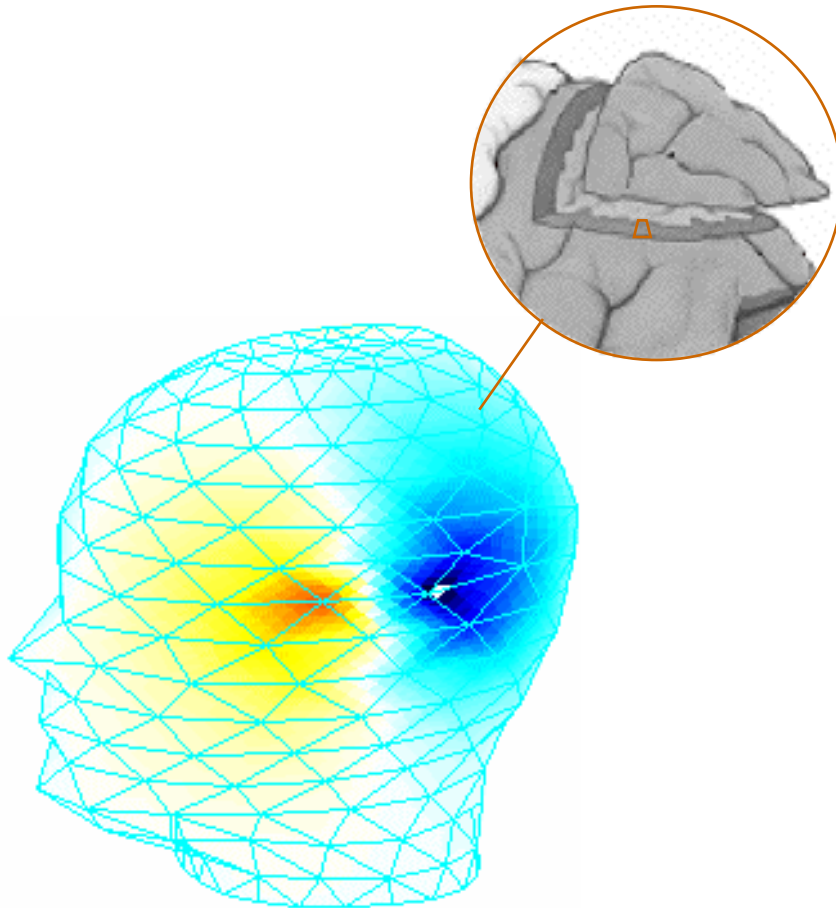


main DCM evolution parameters:

- action potential firing threshold + ensemble PSP spread
- synaptic time constants + axonal propagation delays
- effective coupling strengths + modulatory effects

Neural ensembles dynamics

the observation mapping



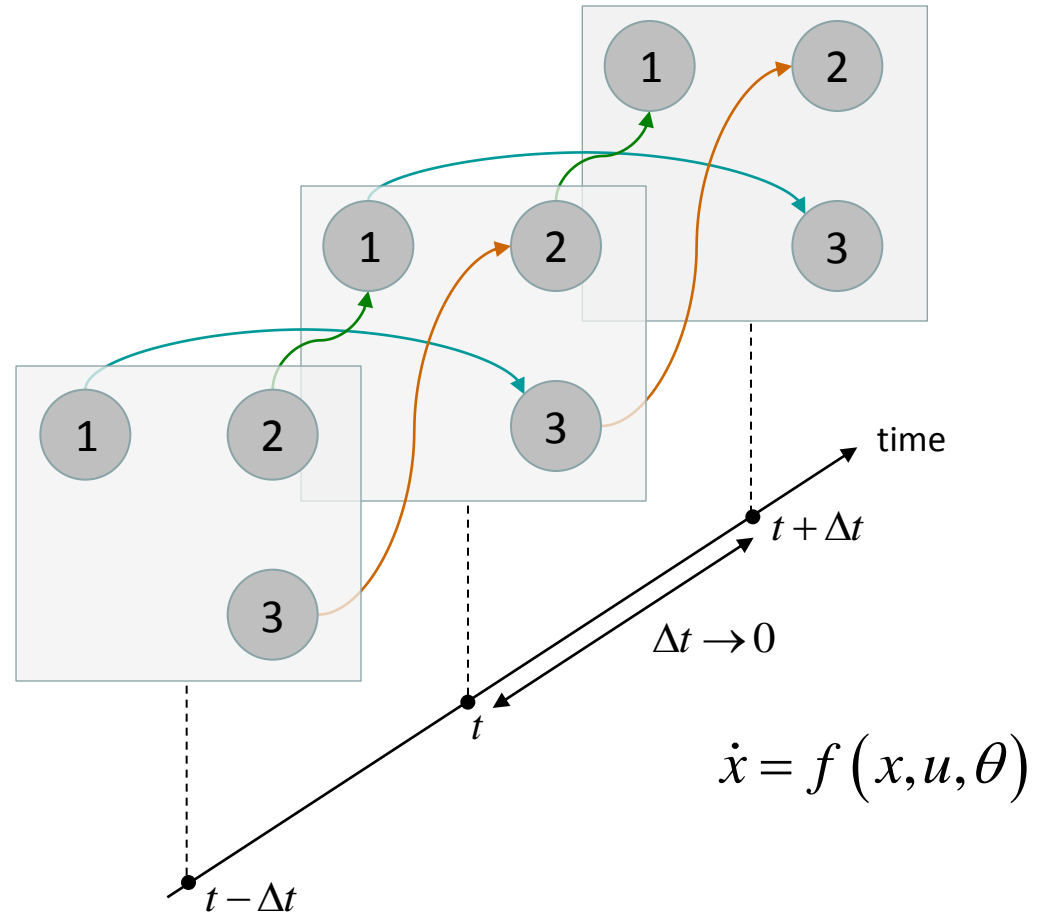
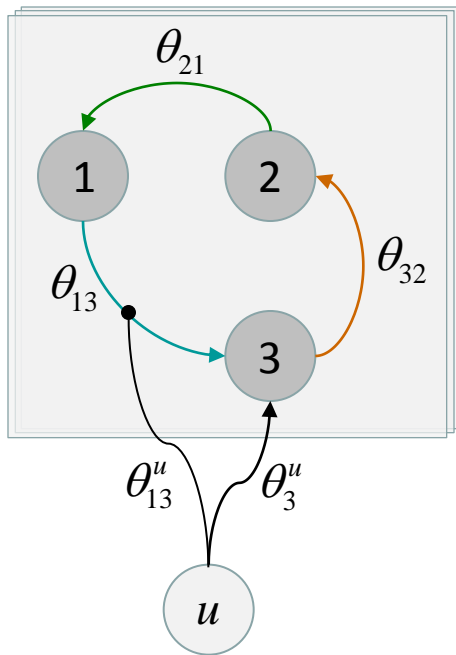
main DCM observation parameters:

- sources location/orientation (ECD) or spatial profile (distributed responses)
- relative contribution of cortical layers to measured signal

Neural ensembles dynamics

a note on causality

$$u \xrightarrow{\theta} x \xrightarrow{\varphi} y$$

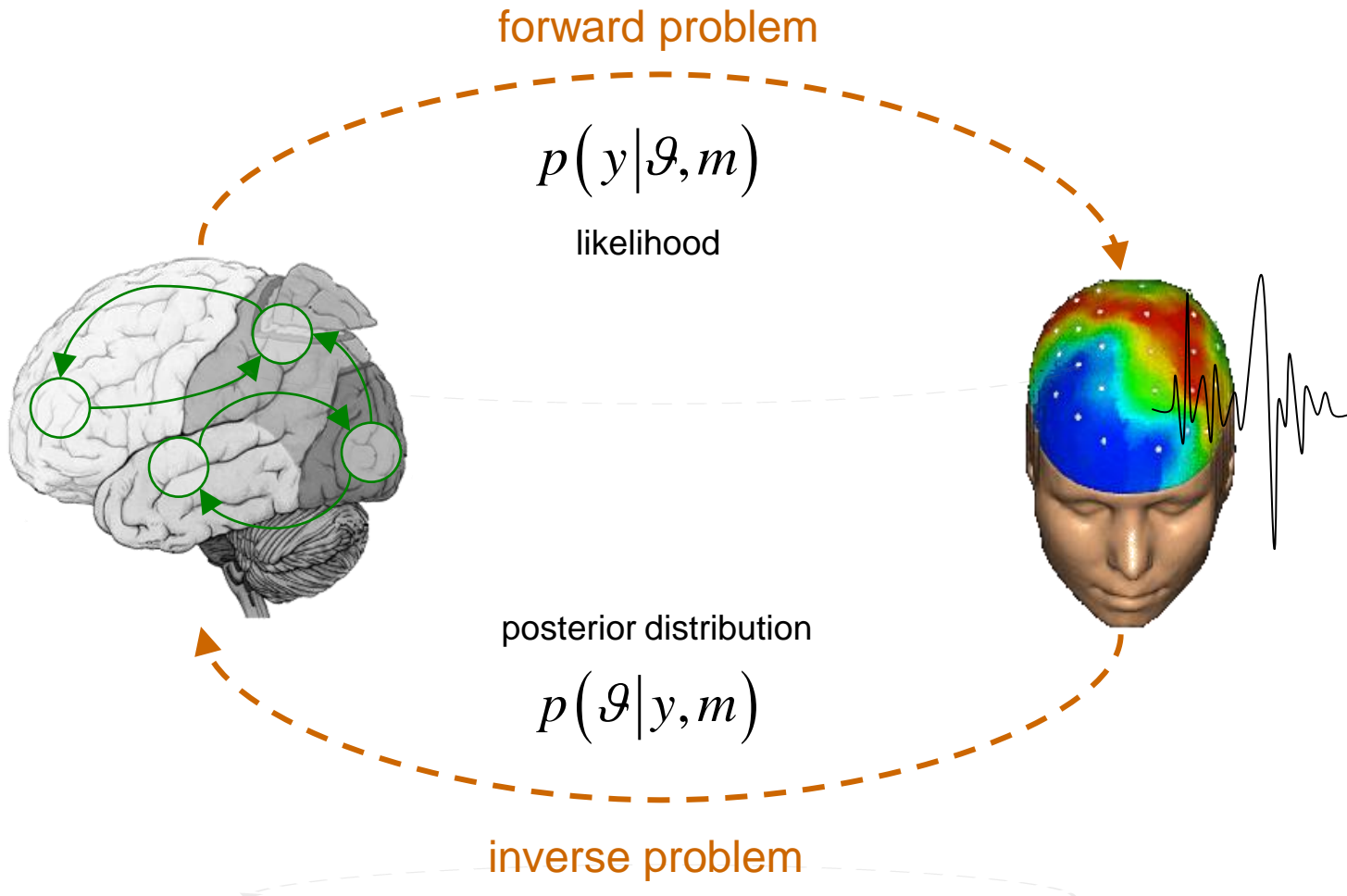


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Bayesian inference

forward and inverse problems



Bayesian inference

deriving the likelihood function

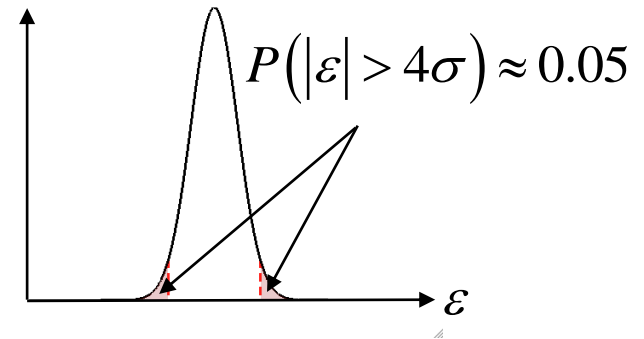
- Model of data with unknown parameters:

$$y = \tilde{g}(\vartheta) \quad \text{e.g., GLM: } \tilde{g}(\vartheta) = X\vartheta$$

- But data is noisy: $y = \tilde{g}(\vartheta) + \varepsilon$

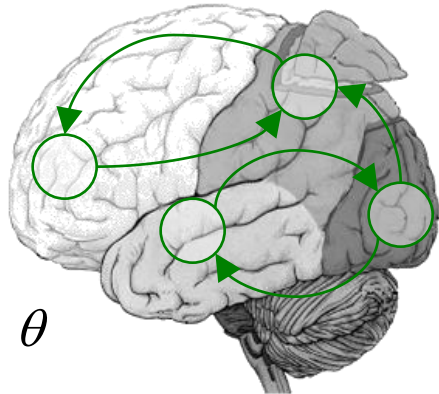
- Assume noise/residuals is 'small':

$$p(\varepsilon) \propto \exp\left(-\frac{1}{2\sigma^2}\varepsilon^2\right)$$



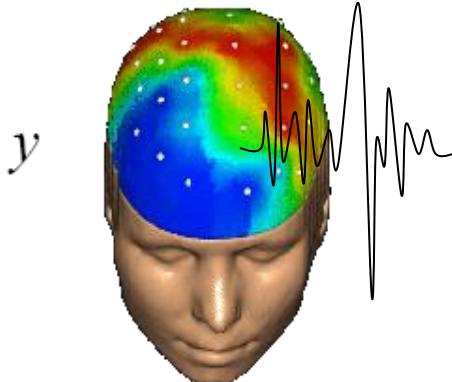
→ Distribution of data, given fixed parameters:

$$p(y|\vartheta) \propto \exp\left(-\frac{1}{2\sigma^2}(y - \tilde{g}(\vartheta))^2\right)$$



θ

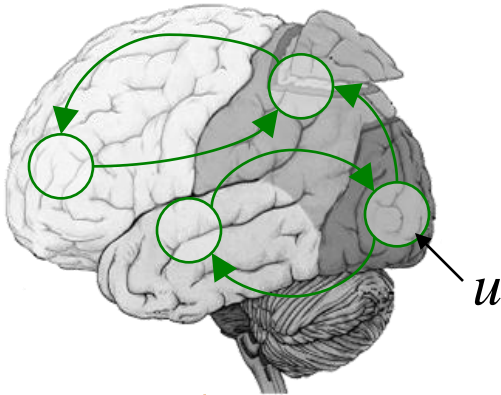
\approx



y

Bayesian inference

likelihood and priors



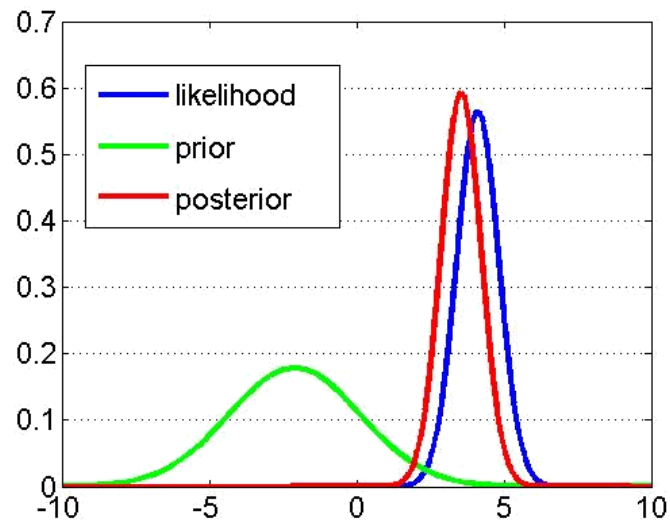
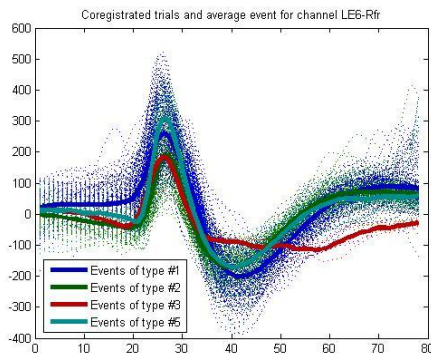
likelihood $p(y|\mathcal{G}, m)$

prior $p(\mathcal{G}|m)$

posterior $p(\mathcal{G}|y, m) = \frac{p(y|\mathcal{G}, m) p(\mathcal{G}|m)}{p(y|m)}$

generative model m

y



Bayesian inference

zooming in the VB algorithm

measured data

specify generative forward model
(with prior distributions of parameters)

Variational Bayesian (VB) algorithm

iterative procedure:

1. compute model response using current set of parameters
2. compare model response with data
3. improve parameters, if possible

1. posterior distributions of parameters
2. model evidence (*free energy*)

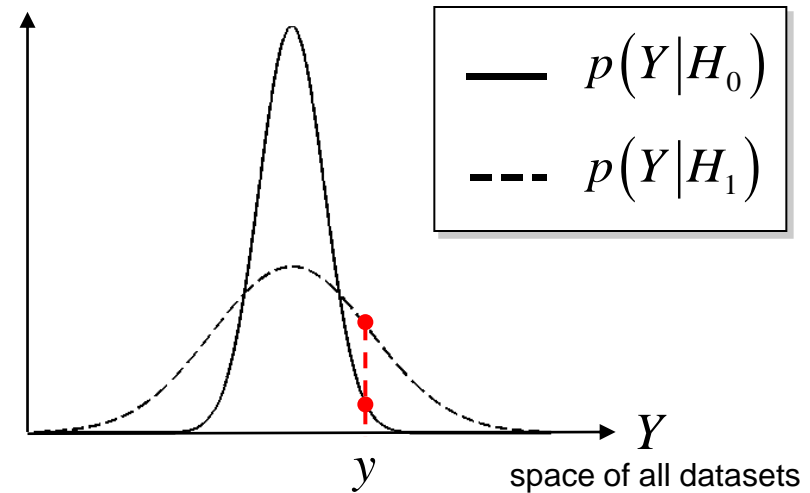
Frequentist versus Bayesian inference

testing point hypotheses

- define the null and the alternative hypothesis *in terms of priors*, e.g.:

$$H_0 : p(\theta|H_0) = \begin{cases} 1 & \text{if } \theta = 0 \\ 0 & \text{otherwise} \end{cases}$$

$$H_1 : p(\theta|H_1) = N(0, \Sigma)$$



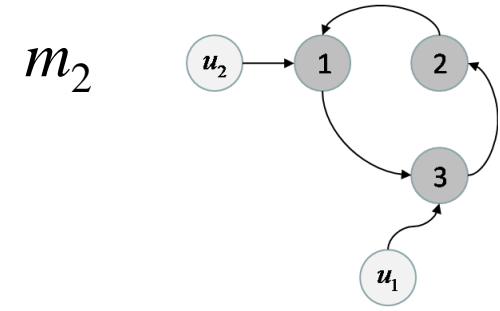
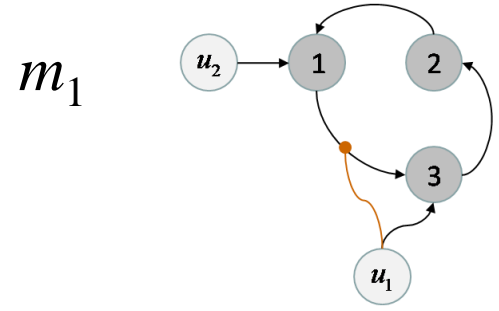
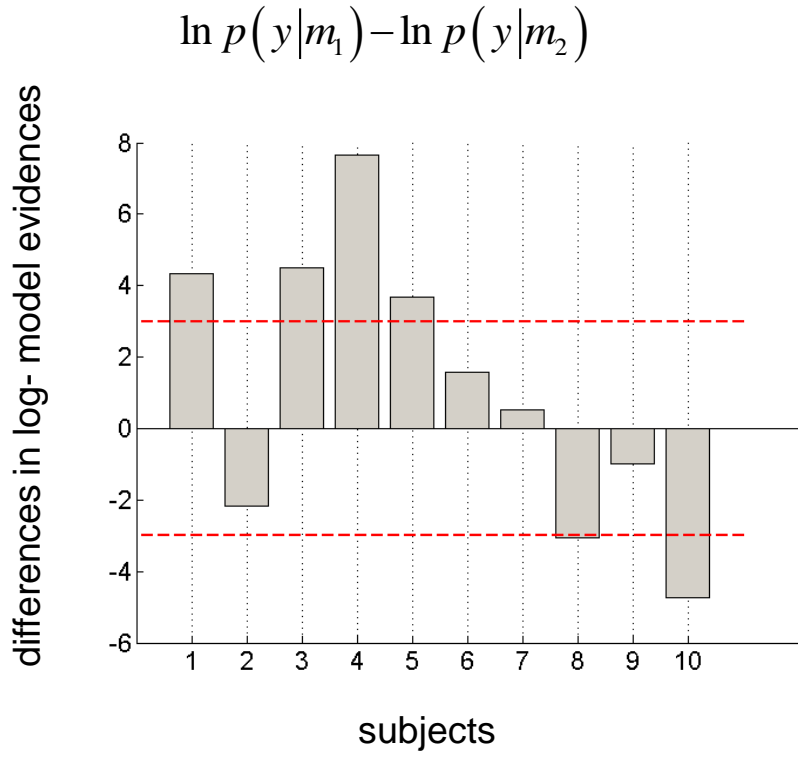
- apply decision rule, i.e.: if $\frac{P(H_0|y)}{P(H_1|y)} \leq 1$ then reject H0

- **Savage-Dickey ratios** (nested models, i.i.d. priors):

$$p(y|H_0) = p(y|H_1) \frac{p(\theta = 0|y, H_1)}{p(\theta = 0|H_1)}$$

Bayesian inference

model comparison for group studies



fixed effect

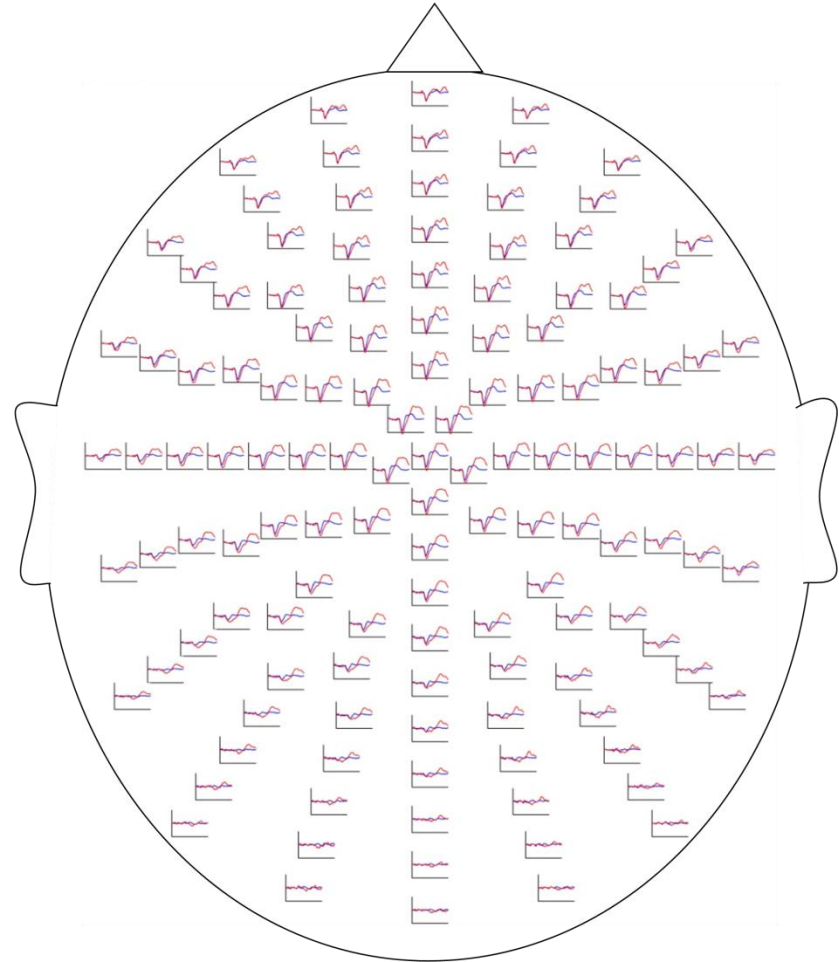
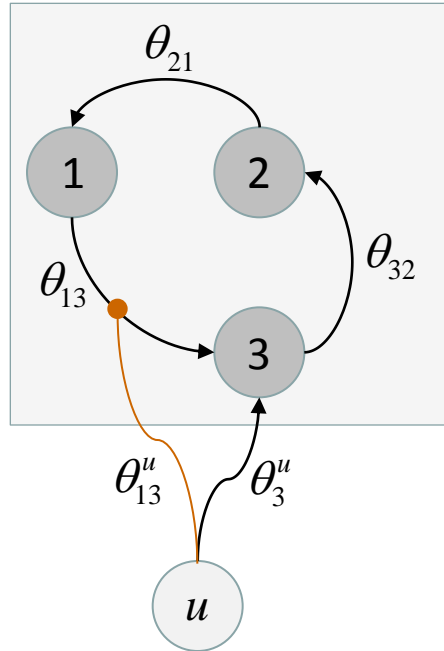
assume all subjects correspond to the same model

random effect

assume different subjects might correspond to different models

Bayesian inference

key DCM parameters



$(\theta_{21}, \theta_{32}, \theta_{13})$ state-state coupling

θ_3^u input-state coupling

θ_{13}^u input-dependent modulatory effect

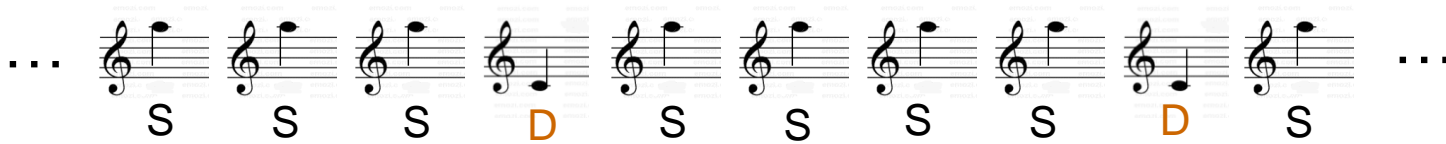
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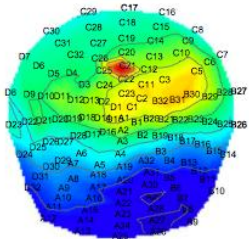
Conclusion

back to the auditory mismatch negativity

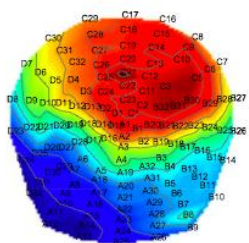
sequence of auditory stimuli



standard condition (S)

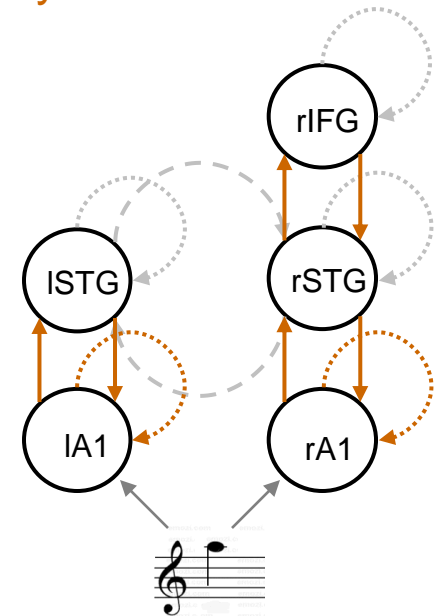
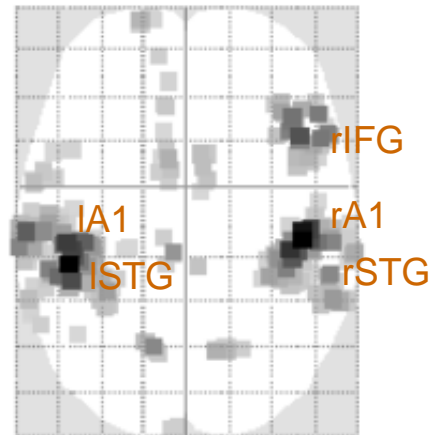


deviant condition (D)



t ~ 200 ms

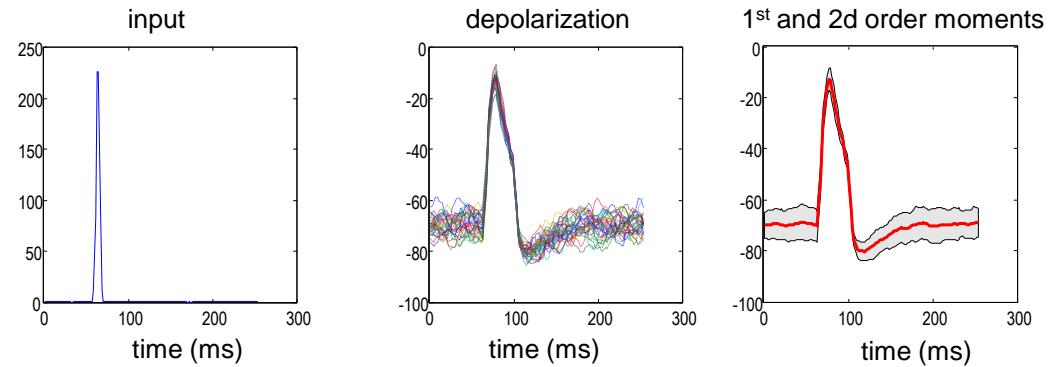
**S-D: reorganisation
of the connectivity structure**



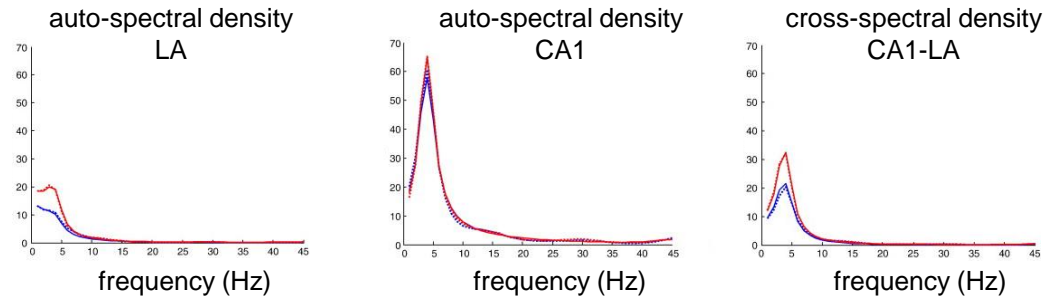
Conclusion

DCM for EEG/MEG: variants

- second-order mean-field DCM

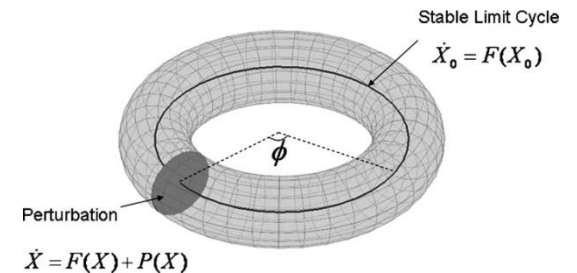
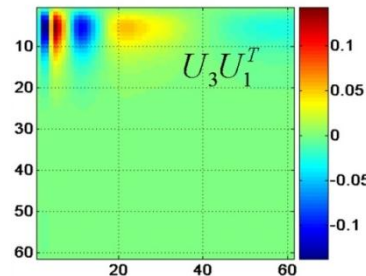


- DCM for steady-state responses



- DCM for induced responses

- DCM for phase coupling



Many thanks to:

Karl J. Friston (London, UK)
Klaas E. Stephan (Zurich, Switzerland)
Stefan J. Kiebel (Leipzig, Germany)

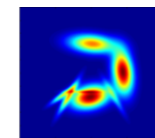
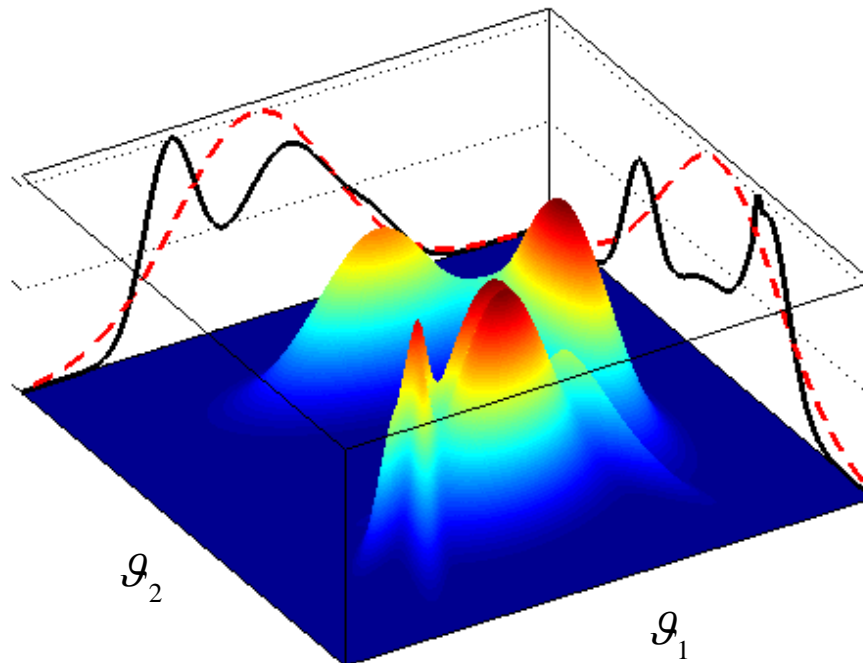
Bayesian inference

the variational Bayesian approach

$$\ln p(y|m) = \underbrace{\langle \ln p(\mathcal{G}, y|m) \rangle_q}_{\text{free energy}} + S(q) + D_{KL}(q(\mathcal{G}); p(\mathcal{G}|y, m))$$

free energy : functional of q

approximate (marginal) posterior distributions: $\{q(\mathcal{G}_1), q(\mathcal{G}_2)\}$



$p(\mathcal{G}_1, \mathcal{G}_2 | y, m)$

— $p(\mathcal{G}_1 \text{ or } 2 | y, m)$

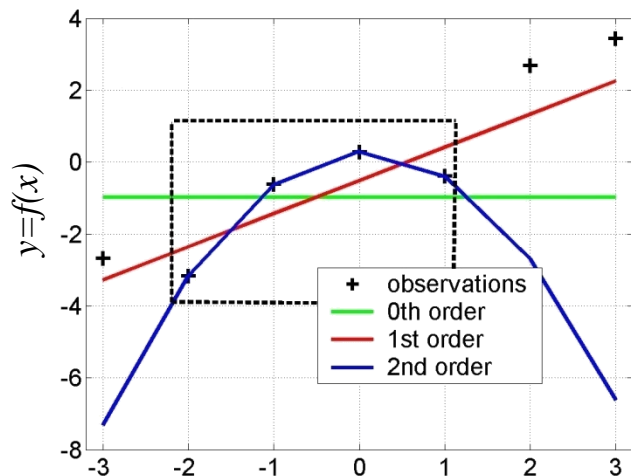
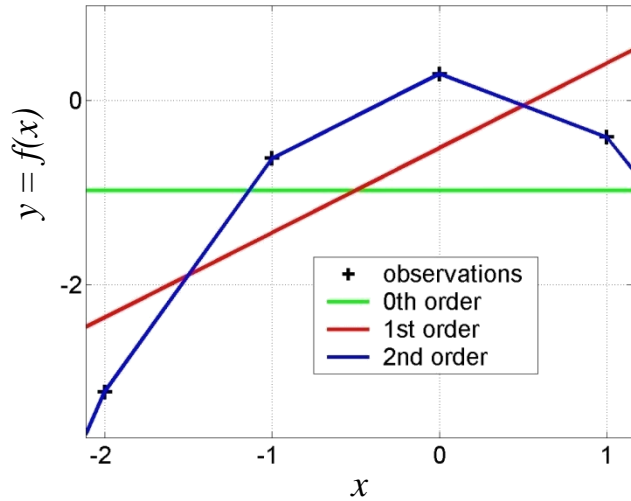
- - - $q(\mathcal{G}_1 \text{ or } 2)$

Bayesian inference

model comparison

Principle of parsimony :

« plurality should not be assumed without necessity »



Model evidence:

$$p(y|m) = \int p(y|\mathcal{G}, m) p(\mathcal{G}|m) d\mathcal{G}$$

“Occam’s razor” :

